POINTWISE IDENTIFICATION OF THERMOPHYSICAL CHARACTERISTICS

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The interior inverse problem of heat conduction for determining the variable coefficient of thermal conductivity and the specific volume heat capacity is studied.

The state vector of a thermal system cannot be determined without reliable data on the thermophysical characteristics (TPC) of the constituent material of the object studied. Such data can be obtained by solving the interior inverse problems of heat conduction (IPH).

The construction of a correct computational algorithm for solving the interior IPH in principle requires information on the nature of the changes of the TPC. Such information, however, is either lacking or is of a very approximate character. Because of this, the polynomial representation of the TPC followed by a search for the constant coefficients of the polynomials approximating the dependences sought, is most widely used. This approach for identifying the coefficients of thermal conductivity of materials using the method of optimal filtering was used in [1], where the constant coefficients of the indicated polynomials were found with quite high accuracy.

The desirability of approximating the characteristics sought by polynomials or some other method (for example, splines) depends to a very large extent on the error of such an approximation, which in its turn is determined by the number of observation points. In addition, increasing the accuracy of the approximation (raising the order of the polynomial) can cause a loss of stability of the estimates obtained in the solution of the IPH. For this reason, it is often better to use the identification which we used previously for determining the boundary conditions of heat transfer [2] and which does not require a preliminary approximation of the dependences sought (separate values of the functions sought are determined at each moment in time, and the dependence being identified is constructed on the basis of these values). We shall call it a pointwise identification.

We shall study below the pointwise identification of the TPC, when the values of the parameters sought are found at each of the nodes of a spatial grid. The method of optimal dynamic filtering, more precisely, its iterative and noniterative modifications, is used.

In this approach the uniqueness of the solution can be guaranteed, if the number of measurements is equal to the number of nodes in the grid. This, naturally, requires a larger computer memory and a faster computer, which cannot always be obtained.

To lower the indicated requirements it is proposed that each time step only one or several values of the function sought, corresponding in some definite manner to the temperatures chosen at each step, be obtained. The method for selecting these temperatures is established by prior study, in the process of which the boundary conditions, the form of the region, and their effect on the temperature field are analyzed. Depending on the nature of the process and the forecasted temperature distribution, in the presence of a sharp nonuniformity of the temperature field some average integral temperatures (from a number of subregions with relatively uniform distribution of temperatures in them) and in the presence of a uniform temperature field the average integral temperature over the entire volume of the body can be used as the determining parameters. The first case requires several (based on the number of subregions) observation points; the second case requires only one observation point. As a result of the solution the dependences $T_{av}(\tau)$, $\lambda(\tau)$ and (or) $C_V(\tau)$, which enable constructing the characteristics sought $\lambda(T)$ or $C_V(T)$.

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The two described approaches to the solution of the interior IPH were compared for test problems. In the first case the characteristic sought was represented as $\lambda(T)=\sum_{m=0}^{N}L_m T^m$.

Then the equation of heat conduction and the boundary conditions (BC) of the second and third kind were written in the finite difference form as follows (to simplify the notation, the one-dimensional case with a uniform grid is considered):

$$\left[-\frac{C_V(T_i)_k}{\Delta\tau}\right](T_i)_{k+1}+\sum_{m=0}^{N}\frac{1}{2^m h^2}\left\{\left[((T_{i+1})_k-(T_i)_k)\times\right.\right.$$
$$\left.\left.\times((T_{i+1})_k+(T_i)_k)^m+((T_{i-1})_k-(T_i)_k)((T_{i-1})_k+(T_i)_k)^m\right]L_m\right\}=\left[-\frac{C_V(T_i)_k}{\Delta\tau}\right](T_i)_k; \tag{1}$$

$$\left[\frac{hC_V(T_b)_k}{2\Delta\tau}\right](T_b)_{k+1}+\frac{1}{h}\sum_{m=0}^{N}\frac{[(T_b)_k+(T_{in})_k]^m[(T_b)_k-(T_{in})_k]}{2^m}L_m$$
$$=q_{k+1}+\left[\frac{hC_V(T_b)_k}{2\Delta\tau}\right](T_b)_k; \tag{2}$$

$$\left[\alpha_{k+1}+\frac{hC_V(T_b)_k}{2\Delta\tau}\right](T_b)_{k+1}+\frac{1}{h}\sum_{m=0}^{N}\left\{\frac{[(T_b)_k+(T_{in})_k]^m}{2^m}\times\right.$$
$$\left.\times[(T_b)_k-(T_{in})_k]L_m\right\}=\alpha_{k+1}(T_m)_{k+1}+\frac{hC_V(T_b)_k}{2\Delta\tau}(T_b)_k. \tag{3}$$

The equations used in the identification of $C_V(T)=\sum_{m=0}^{N}M_m T^m$ can be written down in an analogous manner. The expressions for forecasting the coefficients sought in the problems studied have the form $(\hat{L}_m)_{k+1/k}=(\hat{L}_m)_{k/k}$ and $(\hat{M}_m)_{k+1/k}=(\hat{M}_m)_{k/k}$ with $m=0, 1, 2, \ldots, N$, and instead of the unknown temperatures $(T_i)_{k+1}$, and instead of the unknown temperatures $(T_i)_{k+1/h}$ entering into the coefficients in front of L_m and M_m the forecast of these temperatures from the preceding time step $(T_i)_{k+1/h}$ (or the result of the preceding iteration in the iterative filter) is used.

The algorithm of pointwise identification presupposes that the starting finite difference model of the thermal system is first transformed. In identifying $\lambda(T)$ Eqs. (1)-(3) are converted into the form

$$\left[-\frac{C_V(T_i)^{(l)}_{k+1/k+1}}{\Delta\tau}\right](T_i)^{(l+1)}_{k+1/k+1}+\left[\frac{(T_{i+1}-T_i)(T_{i+1}+T_i)}{2h^2}+\right.$$
$$\left.+\frac{(T_{i-1}-T_i)(T_{i-1}+T_i)}{2h^2}\right]^{(l)}_{k+1/k+1}\lambda(T_i)^{(l+1)}_{k+1/k+1}=\left[-\frac{C_V(T_i)^{(l)}_{k+1/k+1}}{\Delta\tau}\right](T_i)^{(S)}_{k/k}; \tag{4}$$

$$\frac{hC_V(T_b)^{(l)}_{k+1/k+1}}{2\Delta\tau}(T_b)^{(l)}_{k+1/k+1}+\left[\left(\frac{T_b-T_{in}}{h}\right)^{(l)}_{k+1/k+1}\right]\lambda(T_b)^{(l+1)}_{k+1/k+1}=q_{k+1}+\left[\frac{hC_V(T_b)^{(l)}_{k+1/k+1}}{2\Delta\tau}\right](T_b)^{(S)}_{k/k}; \tag{5}$$

$$\left[\alpha_{k+1}+\frac{hC_V(T_b)^{(l)}_{k+1/k+1}}{2\Delta\tau}\right](T_b)^{(l+1)}_{k+1/k+1}+$$
$$+\left[\left(\frac{T_b-T_{in}}{h}\right)^{(l)}_{k+1/k+1}\right]\lambda(T_b)^{(l+1)}_{k+1/k+1}=\alpha_{k+1}(T_c)_{k+1}+\frac{hC_V(T_b)^{(l)}_{k+1/k+1}}{2\Delta\tau}(T_b)^{(S)}_{k/k}, \tag{6}$$

and in the identification of $C_V(T)$ they are converted to the form

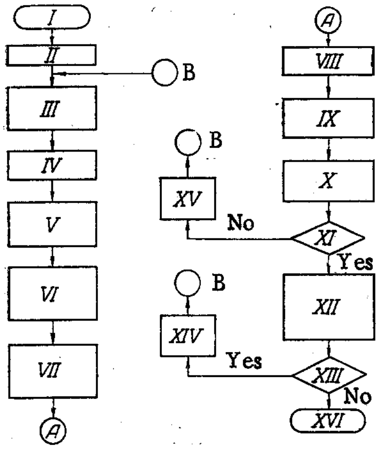

Fig. 1

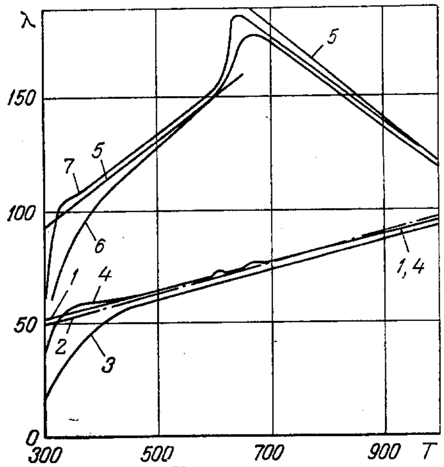

Fig. 2

Fig. 1. Block diagram of an iterative filter ($\varepsilon(\beta)$ is the threshold value; β is the error in the starting data; j is the number of iterations performed; k is the time): I) start; II) input of starting data; III) calculation of the coefficients in the heat-conduction equation; IV) calculation of the transfer matrices; V) calculation of the forecast of the temperature field vector; VI) calculation of the forecast of the vector of parameters being identified; VII) calculation of the covariation matrix of forecast errors; VIII) calculation of the weight matrix; IX) calculation of the estimate of identified parameters; X) calculation of the estimate of the temperature field vector; XI) $-\|\Delta Y\| \leqslant \varepsilon(\beta)$; XII) calculation of the covariation matrix of the refined error estimate; XIII) $\tau < \tau_k$; XIV) $\tau + \Delta\tau$; XV) $j + 1$; and, XVI) end.

Fig. 2. Identification of the coefficient of thermal conductivity, λ, W/m·deg; T, °C.

$$\left[\frac{\lambda(T_{i+1},\,T_i)^{(l)}_{k+1/k+1}}{h^2}\right](T_{i+1})^{(l+1)}_{k+1/k+1}+\left[\frac{\lambda(T_{i-1},\,T_i)^{(l)}_{k+1/k+1}}{h^2}\right](T_{i-1})^{(l+1)}_{k+1/k+1}-\left[\frac{\lambda(T_{i-1},\,T_i)^{(l)}_{k+1/k+1}+\lambda(T_{i-1},\,T_i)^{(l)}_{k+1/k+1}}{h^2}\right]\times$$
$$\times(T_i)^{(l+1)}_{k+1/k+1}-\left[\frac{1}{\Delta\tau}\;(T_i)^{(l)}_{k+1/k+1}\right][(T_i)^{(l)}_{k+1/k+1}-(T_i)^{(S)}_{k/k}]C_V(T_i)^{(l+1)}_{k+1/k+1}=0;$$
$$\left[\frac{\lambda(T_{in},\,T_b)^{(l)}_{k+1/k+1}}{h}\right](T_b)^{(l+1)}_{k+1/k+1}-\left[\frac{\lambda(T_{in},\,T_b)^{(l)}_{k+1/k+1}}{h}\right]\times$$
$$\times(T_{in})^{(l+1)}_{k+1/k+1}+\frac{h[(T_b)^{(l)}_{k+1/k+1}-(T_b)^{(S)}_{k/k}]}{2\Delta\tau}C_V(T_b)^{(l+1)}_{k+1/k+1}=q_{k+1}; \qquad (8)$$
$$\left[\frac{\lambda(T_{in},\,T_b)^{(l)}_{k+1/k+1}}{h}+\alpha_{k+1}\right](T_b)^{(l+1)}_{k+1/k+1}-\left[\frac{\lambda(T_{in}\,T_b)^{(l)}_{k+1/k+1}}{h}\right]\times$$
$$\times\;(T_{in})^{(l+1)}_{k+1/k+1}+\frac{h[(T_b)^{(l)}_{k+1/k+1}-(T_b)^{(S)}_{k/k}]}{2\Delta\tau}C_V(T_b)^{(l+1)}_{k+1/k+1}=\alpha_{k+1}(T_m)_{k+1}. \qquad (9)$$

In the expressions (4)-(9), S is the number of iterations performed at the k-th step and l is the running iteration at the (k + 1)-st step.

The block diagram of the computational algorithm of the iterative filter for solving interior IPH is shown in Fig. 1.

To accelerate the convergence of the solution process, the device or reducing the diagonal elements of the coratiation matrix of evaluation errors to the starting values was used. This device was carried out after a definite number of time steps or at each step.

The regularity of the solutions of the interior IPH was studied in a practical manner. In so doing the computational aspects of the algorithms (iterative and noniterative filters), the nature of the identification made (with approximation of the dependences sought and pointwise identification), effect of the time steps, the error in the measurements and initial approximations, and the location of the points at which the temperature was measured were taken into account.

A test interior IPH for determining $\lambda(T)$ for the material of a plate 0.2 m thick with boundary conditions of the second kind $\left.\frac{\partial T}{\partial x}\right|_{x=0}=0$ on one surface and of the third kind $\lambda(T)\left.\frac{\partial T}{\partial x}\right|_{x=L}=\alpha\,(T-T_m)$ on the other was solved. The following values were used: $C_V = 3\cdot10^6$ J/m$^3\cdot$deg; $T_c = 1400$°C; $\alpha = 300$ W/(m$^2\cdot$deg). The plate was uniformly divided into ten sections with h = 0.02 m, and a time step of $\Delta\tau = 20$ sec was chosen. The temperature was measured at the 10-th node; in addition, the rms value of the noise in "measurements" $\sigma = 0.5\%$ of T_{max}.

The results of the solution are presented in Fig. 2, where the curve 1 shows the nominal value of the characteristics sought ($\lambda = 45 + 0.02$ T) and curve 2 shows the identification $\lambda(T)$ with the polynomial approximation of this dependence. Curves 3 and 4 are the pointwise identification using noniterative and iterative filters, respectively. The initial estimates were chosen as $\hat{\lambda}(T_{av})_{0/0}=20$; $\hat{a}_{0/0}=10$; $\hat{b}_{0/0}=0{,}08$. From a comparison of curves 2 and 4 it is evident that the process of pointwise identification has a much higher rate of convergence and accuracy of the solutions obtained (curve 4 practically coincides with the polynomial approximation already at the fourth step). The only exception is a small initial section, which is generally characteristic for the filtering method.

The stability of the estimates obtained was also studied for examples of test problems but for characteristics which were made artificially more complicated, when the solution is most likely to fall into the zone of possible instability. Thus characteristics having a sawtooth form and a trigonometric form, and functions with discontinuities were studied as the function sought.

We shall examine the definition of the coefficient $\lambda(T)$, having a finite discontinuity of the first kind (curve 5):

$$\lambda(T)=\begin{cases}30+0.2\,T; & T<650^\circ\mathrm{C};\\ 320-0.2\,T; & T\geqslant 650^\circ\mathrm{C}.\end{cases}$$

From a comparison of the results obtained, with the help of noniterative (curve 6) and iterative (curve 7) filters (with $\hat{\lambda}(T_{av})_{0/0} = 50$), from curve 5 it is evident that even in the zone of discontinuity the estimates repeat quite accurately the starting dependence $\lambda(T)$. The maximum relative error in the temperatures with the identified $\lambda(T)$ did not exceed 0.8%. In the example studied above, the initial approximations of the covariation matrices $P_{0/0}$ were not presented, since it was shown previously that the choice of the matrix $P_{0/0}$ does not affect the convergence and stability of the estimates obtained (see, for example, [1]).

The interior IPH for identifying $C_V(T)$ was solved for the same plate for which the dependence $\lambda(T)$ was determined. In this case, boundary conditions of the first kind were given on one of the surfaces ($T_m = 293 + 0.26\,\tau$ °C), a boundary condition of the third kind was given on the other ($\alpha = 25 - 0.01\,\tau$; $T_m = 1400$°C), and the coefficient of thermal conductivity was set equal to $\lambda(T) - 43.49 + 10.61\cdot10^{-3}\,T - 23.02\cdot10^{-6}\,T^2$.

The dependence $C_V(T)$ used to solve the direct problem, i.e., for obtaining "measurements," is presented in Fig. 3 (curve 1). The "measurements" were performed with a periodicity of $\Delta\tau = 37.5$ sec at the fourth and tenth nodes ($\sigma = 0.5\%$ of T_{max}). The initial estimates were chosen as $C_V(T_{av})_{0/0} = 2\cdot10^6$. The results of the identification of $C_V(T)$ are presented in the same figure (curve 2). To evaluate the stability of the solution the problem was made artifically more complicated — the dependence $C_V(T)$ was represented in the form (curve 3)

$$C_V(T)=\begin{cases}1.5\cdot10^6+8\cdot10^3\,T; & T<683^\circ\mathrm{C};\\ 12{,}428\cdot10^6-8\cdot10^3 T; & T\geqslant 683^\circ\mathrm{C}.\end{cases}$$

The identified dependence is presented by curve 4.

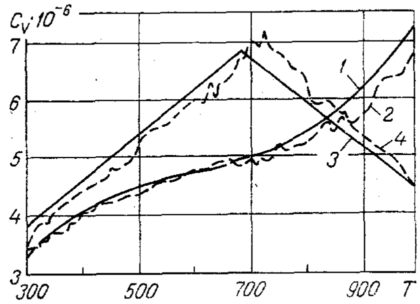

Fig. 3

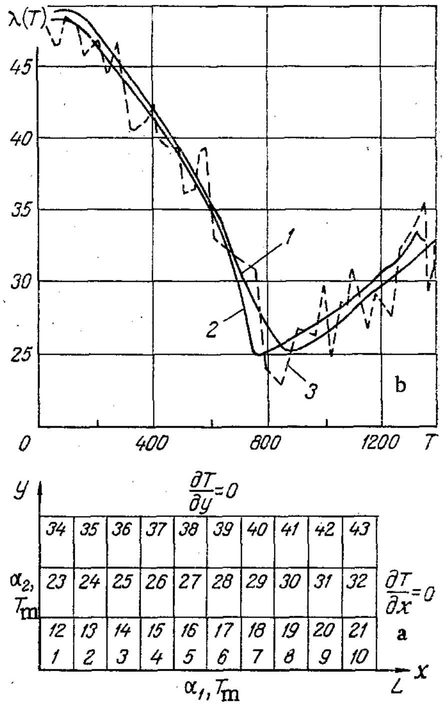

Fig. 4

Fig. 3. Pointwise identification of the specific volume heat capacity. C_V, J/m³·deg.

Fig. 4. Identification of λ(T) of a cooled ingot.

The studies carried out with the solution of the test problems enabled drawing the following conclusions: the location of the nodes at which the temperature is measured and the nature and complexity of the identified TPC do not appreciably affect the regularity of the solutions obtained. At the same time the sizes of the time step and the error in the measurements significantly affect the quality of the estimates. The critical time step [3] must be included in the solution, and the number of iterations in the iterative filter must be matched with the magnitude of the measurement error [2].

The solution of test one-dimensional problems enabled proceeding to the identification of TPC for more complicated objects. Thus, in particular, the coefficient of thermal conductivity of an ingot (45 steel) with a rectangular cross section with continuous pouring of the steel was determined.

Because of the symmetrical nature of the cooling of the ingot, only one-fourth of its cross section (0.51 × 0.0875 m) was studied. The grid model of the object with the conditions of heat transfer on the boundaries is shown in Fig. 4a. Cooling was done first with water (to τ_1) and then with air. In this case the values of the heat transfer coefficients were as follows:

$$\alpha_1 = \begin{cases} 500, & \tau < \tau_1; \\ 150, & \tau \geqslant \tau_1; \end{cases} \quad \alpha_2 = 400, \ \mathrm{W/(m^2 \cdot deg)},$$

while the temperature of the medium was equal to T_m = 24°C. To determine the "measurements" the direct problem with the initial condition $T_0(x, y)$ = 1430°C was solved. The "measurements" were conducted every 30 sec at nodes 13, 19, 27, and 32. The results of the identification of λ(T) with the initial approximations $\hat{T}_{0/0}$ = 1400 and $\hat{\lambda}_{0/0}$ = 45 are presented in Fig. 4b, where curve 1 is the known dependence λ(T) for number 45 steel [4]; curves 2 and 3 are the identified dependences, obtained with the help of the iterative filter with measurement errors of 0.3 and 3% of T_{max}, respectively. The estimates converge to curve 1 already by the fifth time step, and in addition the maximum error does not exceed 8%, indicating not only the good convergence but also the stability of the solutions obtained. The results of the identification of the TPC and the study of the quality of the estimates obtained enable obtaining more accurate and when necessary reliable information on the thermophysical properties of new and little-studied materials, which is especially important for the optimization and intensification of technological heat- and mass-transfer processes in metallurgy and machine building. The studies carried out enable posing the question of the simultaneous identification of several TPC and proceeding to combined IPH, in the course of whose solution the TPC and other conditions of uniqueness, for example, the boundary conditions, geometrical parameters, initial temperature distribution, and so on, are determined in parallel.

NOTATION

λ, coefficient of thermal conductivity; C_V, specific volume heat capacity; T_m, temperature of the medium; T_b and T_{in}, boundary and internal temperatues; α, heat-transfer coefficient) τ, time; $\Delta\tau$, time step; h, spatial step; q, heat flux, σ, rms deviation; and $\|\Delta\bar{Y}\|$, total difference between the measured and predicted temperatures.

LITERATURE CITED

1. Yu. M. Matsevityi, A. V. Multanovskii, and M. Rekada, "Application of the method of optimal filtering to the identification of the coefficient of thermal conductivity," Énerget. Mashinostr., No. 38, 56-61 (1984).
2. Yu. M. Matsevityi and A. V. Multanovskii, Identification in Heat-Conduction Problems [in Russian], Naukova Dumka, Kiev (1982).
3. O. M. Alifanov, Identification of Heat Transfer Processes in Aircraft [in Russian], Mashinostroenie, Moscow (1979).
4. B. E. Neimark (ed.), Physical Properties of Steels and Alloys Used in Power Generation [in Russian], Énergiya, Moscow (1967).

NONLINEAR INVERSE PROBLEM OF RECONSTRUCTING TRANSPORT COEFFICIENTS

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The inverse coefficient problem for the quasilinear heat-conduction equation is solved numerically.

The experimental determination of thermophysical parameters is usually based on the solution of direct problems in the theory of heat conduction, when for fixed properties of the medium the temperature field is found with the help of the theory, and methods for confirming experimentally the theoretical results are created based on the theoretical representations. At high temperatures, however, experimental measurements are difficult to perform, so that the thermophysical properties of the materials are determined using the values of the temperature distribution measured far from the contact surface with the high-temperature flow. These problems, called inverse problems of transfer theory, have become very important in recent years in connection with the extensive possibilities presented by modern computational methods together with the extensive use of computers for rapid determination of the thermophysical parameters.

In most cases, linear mathematical models are used to solve inverse problems of determining the thermophysical parameters. In a wide range of temperature variation, however, the temperature dependence of the thermophysical parameters cannot be ignored, so that these methods obviously suffer from substantial errors. It is natural to base the experimental methods of determining thermophysical parameters on nonlinear mathematical models [1, 2].

In this work we study the problem of determining the nonlinear thermophysical characteristics, the heat capacity, and the coefficient of thermal conductivity for metal cylindrical samples interacting with a high temperature flow by the method of conjugate gradients.

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